A Comparison of the Belief-Adjustment Model and the Quantum Inference Model as Explanations of Order Effects in Human Inference

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Abstract

One of the oldest and most reliable findings regarding human inference is that the order of evidence affects the final judgment. These order effects are non-Bayesian by nature and are difficult to explain by classical probability models. We use the empirical results of two jury decision-making experiments to compare two different models of human belief updating: the belief-adjustment model and the quantum inference model. We also provide evidence to suggest the belief-adjustment model has limited predictive power when accounting for tasks involving extreme evidence whereas the quantum inference model does not.

Keywords: inference; jury decision-making; order effects; recency effects; belief-adjustment model; quantum inference model

Introduction

Human inference provides a rich source of evidence for a non-Bayesian belief updating process. Consider a physician deciding whether a certain patient has an infection or not. The physician first examines the patient and takes a medical history. At this point, the physician has some degree of belief in the presence of the infection. Then the physician orders a laboratory test and revises those beliefs. Now, suppose the physician had proceeded by first administering the laboratory test followed by the physical examination. Would the physician ultimately have the same belief about the infection when the order of information is reversed? Bergus, Chapman, Levy, Ely, et Oppliger (1998) would argue that the order of information, physical examine followed by laboratory test versus laboratory test followed by physical examine, has a significant impact on the physician's final belief in the presence of the infection.

Order of information plays a crucial role in the process of updating beliefs across time (Hogarth & Einhorn, 1992). The presence of order effects makes a classical or Bayesian approach to inference difficult. Specifically, suppose a decision-maker must ascertain the probability that a certain hypothesis, H, is true after seeing two pieces of evidence, X and Y. Classical probability requires $P(X,Y|H) = P(Y,X|H)$, and by Bayes rule we must have $P(H|X,Y) = P(H|Y,X)$. Thus, a simple Bayesian model makes no distinction between different orders of information.

In this paper, we compare two possible explanations for order effects, the belief-adjustment model (Hogarth & Einhorn, 1992) and the quantum inference model (Busemeyer & Trueblood, 2009). The belief-adjustment model accounts for order effects by either adding or averaging evidence. The quantum inference model explains order effects by transforming a state vector with different sequences of operators for different orderings of information.[1] We first examine both models with data collected from a jury decision-making experiment conducted by McKenzie, Lee, et Chen (2002). Then we test both models using new data collected from two new experiments that extend the work of McKenzie et al.

A Jury Decision-Making Experiment

McKenzie et al. conducted two experiments to examine the effects of case order and strength on changes in subjects' confidence ratings. In this study, subjects were asked to read a criminal case concerning a burglarized warehouse and to rate their confidence in the defendant's guilt, G. In the first experiment, one group of participants read a strong prosecution, SP, followed by a weak defense, WD. The other group read the information in the reverse order, the weak defense followed by the strong prosecution. For the second experiment, the first condition was identical to the first condition in experiment 1. However, in the second condition, subjects read a weak prosecution, WP, followed by the weak defense. In both experiments, subjects provided confidence ratings as a number between 0 and 20 before reading either case, after reading the first case, and after reading the second case. A separate group of subjects rated the strength of the prosecution and defense and did not participate in the inference task. By averaging the data from condition one of experiment 1 with condition one of experiment 2 and converting the mean confidence ratings to probabilities, we have the results shown in Table 1.

One of the most interesting aspects of these results is that the weak defense increased confidence in guilt when preceded by the strong prosecution but decreased confidence in guilt when preceded by the weak prosecution. The interpretation of the defense as evidence for guilt when coupled with the strong prosecution and evidence for innocence when coupled with the weak prosecution resists explanation by the standard belief-adjustment model (McKenzie et al., 2002).

[1] We use quantum theory as a mathematical tool and do not attach the physical meaning associated with quantum physics. This type of approach is similar to the use of stochastic processes outside the domain of physics.

Table 1: Probability of Guilt from Experiments 1 and 2

After first case	After second case
Pr(G | SP) = 0.672	Pr(G | SP, WD) = 0.719
Pr(G | WD) = 0.51	Pr(G | WD, SP) = 0.75
Pr(G | WP) = 0.600	Pr(G | WP, WD) = 0.525

An extended version of this model, the minimum acceptable strength model (MAS), uses a variable reference point to model these results (McKenzie et al., 2002). As an alternative to the MAS model, the quantum model uses a series of transformations to explain the phenomena. Before we proceed with fitting the two models, we will outline the belief-adjustment model and the MAS model. We will also provide an intuitive description of the quantum model.[2]

The Belief-Adjustment Model

The belief-adjustment model assumes individuals update beliefs by a sequence of anchoring-and-adjustment processes (Hogarth & Einhorn, 1992). The algebraic description of the model is

$$C_k = C_{k-1} + w_k \cdot (s(x_k) - R) \quad (1)$$

where $0 \leq C_k \leq 1$ is the degree of belief in the defendant's guilt after reading case k, $s(x_k)$ is the strength of case k, R is a reference point, and $0 \leq w_k \leq 1$ is an adjustment weight for case k. Hogarth et Einhorn argue that evidence can be encoded either in an absolute manner or in relationship to the current belief in the hypothesis. If evidence is encoded in an absolute manner and there exists a positive/negative relationship between the evidence and hypothesis, $R = 0$ and $-1 \leq s(x_k) \leq 1$. However, if evidence is encoded in relationship to the current belief, $R = C_{k-1}$ and $0 \leq s(x_k) \leq 1$. Also, Hogarth et Einhorn assume that the adjustment weight w_k depends on the level of current belief and the sign of the difference $s(x_k) - R$. Specifically, if $s(x_k) \leq R$, then $w_k = C_{k-1}$. However, if $s(x_k) > R$, then $w_k = 1 - C_{k-1}$.

Using this information, we can rewrite the belief-adjustment model as either an adding model or an averaging model. The adding model results when information is encoded in an absolute manner and is given by

$$C_k = \begin{cases} C_{k-1} + C_{k-1} \cdot s(x_k), & \text{if } s(x_k) \leq 0 \\ C_{k-1} + (1 - C_{k-1}) \cdot s(x_k), & \text{if } s(x_k) > 0 \end{cases}$$

On the other hand, the averaging model results when information is encoded in relationship to the current belief and is given by

$$C_k = \begin{cases} C_{k-1} + C_{k-1} \cdot (s(x_k) - C_{k-1}), & \text{if } s(x_k) \leq C_{k-1} \\ C_{k-1} + (1 - C_{k-1}) \cdot (s(x_k) - C_{k-1}), & \text{if } s(x_k) > C_{k-1} \end{cases}$$

Rearranging the terms above shows that the current belief is an average of the prior belief and the strength of the new evidence weighted by the prior belief.

[2] Trueblood et Busemeyer (2010) contains a complete mathematical description of the quantum inference model.

The MAS model extends the belief-adjustment model by defining the reference point as a case's minimum acceptable strength (McKenzie et al., 2002). Thus, equation 1 becomes

$$C_k = C_{k-1} + w_k \cdot (s(x_k) - m_{k-1}) \quad (2)$$

where m_{k-1} is the minimum acceptable strength of the previous case and $-1 \leq s(x_k) \leq 1$. Neither the adding or averaging models can predict that a defense would increase confidence in guilt. However, it is possible to select a value for m_{k-1} such that the difference between the strength of the weak defense and m_{k-1} is positive. Therefore, confidence in guilt increases as a result of the weak defense. The downside to the MAS model is the increase in parameters. The adding and averaging models specify a parameter for each case, namely $s(x_k)$. However, the MAS model also needs a minimum acceptable strength parameter for each case; thereby, doubling the number of parameters needed in the original model.

The Quantum Inference Model

There are several reasons for considering a quantum approach to human judgments. First, judgment is not a simple read out from a pre-existing or recorded state, instead it is constructed from the current context and question. Thus, making a judgment changes the context which disturbs the cognitive system. This implies that changes in context produced by the first judgment influence the next judgment resulting in order effects. Therefore, human judgments do not obey the commutative rule of classic probability theory suggesting that classical probability theory is too limited to fully explain various aspects of human judgment and decision-making. Other such phenomena include violations of the sure thing axiom of decision-making (Tversky & Shafir, 1992) and violations of the conjunctive and disjunctive rules of classic probability theory (Gilovich, Griffin, & Kahneman, 2002).

We describe the quantum inference model in terms of the specific jury decision-making task outlined above; however, this model can be extended to any number of hypotheses and pieces of evidence (Busemeyer & Trueblood, 2009). The quantum inference model assumes that a decision-maker can view the two complementary hypotheses, guilty (h_1) and not guilt (h_2), from three different points of view. The first point of view is considered neutral (N) and is associated with the judgment made before either case is read. The second point of view is associated with the prosecution's case (P), and the third point of view is associated with the defense's case (D). The prosecution is assumed to present evidence for guilt (e_1), and the defense is assumed to present evidence for innocence (e_2). Considering all possible combinations of hypotheses and evidence, we have four patterns of the form $h_i \wedge e_j$. These four patterns or joint events define a four dimensional vector space. An individual's beliefs about these events are represented as a four dimensional state vector, ψ, situated within this four dimensional vector space. The three points of view are represented mathematically as three different bases for this vector space. Thus, there are three different vector representations of ψ corresponding to the neutral basis, the prosecution basis, and the defense basis: $\psi_N = \omega$, $\psi_P = \alpha$, and $\psi_D = \beta$. The four dimensional unit column vectors ω, α, and β represent the probability amplitudes for the joint events, $h_i \wedge e_j$, with respect to the different bases, or points of view.[3]

A set of matrix operators act on ψ to transform an individual's beliefs in correspondence with changes of perspective. Specifically, the probability amplitudes for one point of view are transformed into the probability amplitudes for a different point of view by unitary transformations:

$$\alpha = U_{pn} \cdot \omega$$

$$\beta = U_{dn} \cdot \omega.$$

For example, suppose an individual makes a judgment after reading the prosecution and then again after reading the defense. First, the individual sees the prosecution present evidence (e_1) favoring the guilty hypothesis. We project $\psi_P = \alpha$ onto the subspace corresponding to the evidence:

$$\psi_P = \begin{bmatrix} \alpha_{h_1 \wedge e_1} \\ \alpha_{h_1 \wedge e_2} \\ \alpha_{h_2 \wedge e_1} \\ \alpha_{h_2 \wedge e_2} \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha_{h_1 \wedge e_1} \\ 0 \\ \alpha_{h_2 \wedge e_1} \\ 0 \end{bmatrix}.$$

We then normalize this projection to ensure that the length of the new state vector, $(\psi_P \mid e_1)$, equals one. When the individual is questioned about the conditional probability of guilt given the prosecution, the revised state is projected onto the guilty subspace. With the presentation of the defense, the revised state vector, $(\psi_P \mid e_1)$, is transformed from its vector representation associated with the prosecution basis to its vector representation associated with the defense basis by $U_{dp} = U_{dn} \cdot U_{pn}^{\dagger}$.[4] Now, we project our revised state onto the e_2 subspace since the defense is assumed to present evidence for innocence. Again, we normalize the state vector and project it onto the guilty subspace to calculate the conditional probability of guilt given the prosecution followed by the defense. Order effects arise because the unitary transformations are non-commutative. Figure 1 provides a schematic for the different sequences of transformations used for the different case orderings.

The model parameters define the specific matrix operators, U_{pn} and U_{dn}, used to transform one representation of ψ to another. We define a parameter for each case. So, there is a parameter associated with the strong prosecution, weak defense, and weak prosecution. Thus, to model the data collected by McKenzie et al. the quantum model uses three parameters.

Fitting the Data

We fit both the MAS model and the quantum model to the six probabilities shown in Table 1. Both models capture the

[3]Probability amplitudes determine the belief about a particular event. Probabilities are calculated from probability amplitudes by taking the modulus of the amplitude and squaring.

[4]$U^{\dagger}$ is the conjugate transpose of U. For unitary matrices, $U^{\dagger}$ is also the inverse of U.

$$\begin{tikzcd} \psi_P & \psi_N \arrow[r, bend right, "U_{dn}"'] & \psi_D \arrow[ll, bend right, "U_{pd}"'] \end{tikzcd}$$

$$\begin{tikzcd} \psi_P \arrow[rr, bend left, "U_{dp}"] & \psi_N \arrow[l, bend left, "U_{pn}"] & \psi_D \end{tikzcd}$$

Figure 1: Transformations for different case orderings: defense followed by prosecution (top) and prosecution followed by defense (bottom).

qualitative properties of the data. Namely, $Pr(G \mid SP) < Pr(G \mid SP, WD)$ and $Pr(G \mid WP) > Pr(G \mid WP, WD)$. The quantum model fit the data with three parameters with the sum of squared error (SSE) equal to 0.00056; whereas, the MAS model fit the data with four parameters with the SSE = 0.0022. The SSE for the models is very small since we are examining differences in probabilities. Of the four parameters in the MAS model, three were associated with the minimum acceptable strength. The fourth parameter was the gradient parameter of a logistic function used map the average independent strength ratings from participants into the interval $[-1, 1]$. If we compare the SSE from the quantum model to the SSE from the MAS model, we see that the SSE for the quantum model is much smaller than the SSE for the MAS model:

$$\frac{SSE_{MAS}}{SSE_{qt}} = \frac{0.0022}{0.00056} = 3.93.$$

Furthermore, since the quantum model has less parameters and a smaller SSE, it will have a lower BIC value than the MAS model.

Experiment 1: Extending McKenzie

McKenzie et al. did not examine all possible combinations of case strength and order. Assuming there are only two possible strengths, weak and strong, there are twelve total possible conditional probability judgments that can be made (see Table 2). Thus, we designed a new experiment to collected data for these twelve probabilities. Participants in this new study read eight different scenarios involving a defendant standing trial for either robbery, larceny, or burglary. Each participant was placed into one of eight conditions for each scenario. These eight conditions arise from the eight possible sequential judgments that can be made when taking into consideration order and case strength (e.g. weak prosecution followed by strong defense). Participants were placed in a different condition for each crime so they would experience all eight conditions by the end of the experiment. The participants reported the likelihood of the defendant's guilt before reading either case, after the first case, and after the second case.

Table 2: Conditional Probabilities for Jury Task

After first case	After second case	
Pr(G | WP)	Pr(G | WP, WD)	Pr(G | WP, SD)
Pr(G | SP)	Pr(G | SP, WD)	Pr(G | SP, SD)
Pr(G | WD)	Pr(G | WD, WP)	Pr(G | WD, SP)
Pr(G | SD)	Pr(G | SD, WP)	Pr(G | SD, SP)

Method

Participants in the study were 299 undergraduate students from Indiana University who received experimental credit for introductory psychology courses. For each scenario, there were approximately 38 participants in each each condition. All stimuli were presented on a computer and students entered their responses using the computer keyboard. For each scenario, participants were asked to imagine that they were jurors on the trial. They were also told that in each crime, the defendant was arrested after the police received an anonymous tip. One of the eight scenarios was directly patterned after the crime used by McKenzie et al. Likelihood of the defendant's guilt was reported on a continuous scale from 0 to 1 with 0 = certain not guilty, 0.5 = equally likely, and 1 = certain guilty.

Results

Eight of the 299 participants were excluded from the analyses because the majority of their initial ratings (before being presented with the prosecution or defense) were 0. These participants most likely assumed a literal interpretation of 'innocent until proven guilty'.

We first analyzed each scenario alone, and our analysis revealed a prevalence of recency effects. These effects arise when decision-makers place disproportionate importance on recent evidence (e.g. $Pr(G \mid SP, SD) < Pr(G \mid SD, SP)$). For each crime, there were four defense-prosecution pairs (SD v. SP, SD v. WP, WD v. SP, and WD v. WP) that could exhibit order effects. A two sample t-test showed the majority of pairs exhibited a significant recency effect ($p < 0.05$).

Since the scenarios were designed to be very similar, we reanalyzed the data by collapsing across all eight scenarios. A two sample t-test showed a significant recency effect for each of the four defense-prosecution pairs ($p < 0.001$).

Fitting the Data

The presence of recency effects in this new data set confirms earlier findings and provides the largest data set so far for comparing models that explain recency effects. Hogarth et Einhorn discovered that recency effects are prevalent in simple, step-by-step tasks with short series of evidence. Furthermore, there is evidence of recency effects in studies involving mock trials (Furnham, 1986 ; Walker, Thibaut, & Andreoli, 1972). Unlike the study conducted by McKenzie et al., none of the cases in this study caused a reversal in likelihood judgment when paired with opposing cases of different strengths. This might be due to the use of a standard numeric measure instead of a 21-point confidence scale. There is research showing that standard numeric measures can be insensitive to some judgment phenomena (Windschitl & Wells, 1996).

Since the data does not exhibit the effects found by McKenzie et al., we can fit the standard belief-adjustment model instead of the extended MAS model. We fit the averaging model, the adding model, and the quantum inference model to the mean likelihood of guilt for the eight different crimes as well as the averaged data. All three models use four parameters to fit the twelve data points associated with each crime. These parameters were fit by minimizing the sum of squared error between the data and model predictions. The four parameters used by the averaging and adding models arise from the four case strengths, $s(x_k)$, in equation 1. The four parameters for the quantum model arise from the matrix operators used to transform the state or belief vector. The minimized SSE for all three models are shown in Table 3. From this table, we see that both the adding and quantum models fit better than the averaging model. Also, the quantum model fits slightly better than the adding model in most cases.

Table 3: Model Fits

Crime	Averaging	Adding	Quantum
1	0.0719	**0.0112**	0.0132
2	0.0634	0.0083	**0.0056**
3	0.1185	0.0213	**0.0070**
4	0.0939	0.0156	**0.0127**
5	0.0913	**0.0091**	0.0109
6	0.0656	0.0130	**0.0113**
7	0.0913	0.0217	**0.0089**
8	0.0620	0.0164	**0.0023**
Average	0.0704	0.0059	**0.0058**

Figure 2 shows the model fits for the averaging model and quantum model for the strong defense-weak prosecution pair for the averaged data. From the figure, we see that the quantum model provides a much better fit. Fits for the remaining defense-prosecution pairs are similar.

Experiment 2: Extreme Evidence

To provide even more of a distinction between the quantum model and the belief-adjustment model, we conducted a second jury decision-making experiment involving extreme evidence. In this task, subjects read about an individual on trial for a crime in which the defense had an irrefutable argument. Specifically, the defense stated that the defendant was giving a public lecture when the crime was committed. The prosecution's argument was moderately strong: a witness claimed to

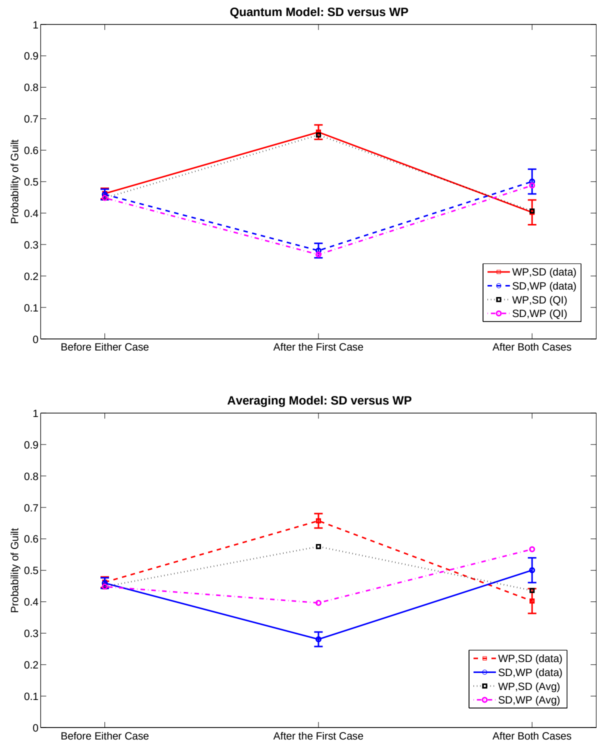

Figure 2: Averaging and quantum model fits to the mean likelihood of guilt for the strong defense-weak prosecution pair. Error bars on the data show the 95% confidence interval.

have seen the defendant near the scene of the crime. It seems reasonable to believe that the probability of guilt after hearing the defense will be near zero. Now, if the prosecution is presented after the defense, it is unlikely that the probability of guilt will increase by much. In terms of the belief-adjustment model, this places tight restrictions on the value of the prosecution strength parameter, $s(x_p)$. To see this, let's examine the adding version of the model:

$$C_p = C_d + (1 - C_d) \cdot s(x_p)$$

where C_p is the evaluation of the guilty hypothesis after hearing the prosecution's case and the irrefutable defense. We might assume the evaluation of the hypothesis after hearing just the defense, C_d, is near zero, say $C_d = \varepsilon_1$. Thus, $s(x_p)$ must also be a near zero, say $s(x_p) = \varepsilon_2$, in order for C_p to remain small:

$$C_p = \varepsilon_1 + (1 - \varepsilon_1) \cdot \varepsilon_2 = \varepsilon_1 + \varepsilon_2 - \varepsilon_1 \cdot \varepsilon_2 \approx 0.$$

Now, suppose the prosecution is presented before the defense. According to the adding model,

$$C_p = C_0 + (1 - C_0) \cdot s(x_p)$$

where C_0 is the evaluation of the guilty hypothesis before hearing either the prosecution or defense. We might assume that $C_0 \approx 0.5$. Thus, we have

$$C_p = 0.5 + 0.5 \cdot \varepsilon_2 \approx 0.5$$

showing the prosecution has little impact on the initial evaluation of the hypothesis. However, it seems unlikely that initial beliefs will be unaltered by the presentation of the prosecution. On the contrary, we might expect this prosecution to be very effective when no prior defense is presented. Essentially, the problem arises from the model's assumption that the strength of the prosecution, $s(x_p)$, is determined independently of other evidence.

This study used 164 undergraduate psychology students. Subjects were placed into one of two conditions corresponding to the two possible case orders: prosecution followed by defense or defense followed by prosecution. Similar to experiment 1, subjects entered responses on a computer and were told that the defendant was arrested after the police received an anonymous tip. Instead of providing the likelihood of the defendant's guilt, subjects were asked to rate their confidence in guilt on the same 21-point scale used by McKenzie et al. Like experiment 1, a significant recency effect was found ($p < 0.023$).

We converted the confidence ratings to probabilities and fit the quantum model and the adding model to the mean of these probabilities. We did not fit the averaging model since experiment 1 shows the adding model outperforms the averaging model. Figure 3 shows the model fits for the two models. Both the quantum model and the adding model use two parameters to fit the data. The SSE for the quantum model was 0.0002; whereas, the SSE for the adding model was 0.0158. By comparing the ratio of the SSE from the two models, we see that the quantum model provides a much better fit to the data:

$$\frac{SSE_{adding}}{SSE_{qt}} = \frac{0.0158}{0.0002} = 79.$$

The standard belief-adjustment model cannot capture dependences between the strength of the prosecution and the irrefutable defense. As a result, the model provides a poor fit to the data. Unlike the belief-adjustment model, the quantum model does not assume individuals combine evidence by simple arithmetic procedures such as adding or averaging. Instead, the quantum model supports the idea that evidence is viewed from different perspectives, and it is these different, or incompatible, points of view that allow the quantum approach to capture the effects of extreme evidence.

Conclusion

One might question the extent to which quantum probability models are rational. Like classic (Kolmogorov/Bayesian) probability theory, quantum theory is based on a coherent set of axioms. Then the question falls back on which set of axioms is most appropriate for an application. For example, models based on the axioms of quantum probability theory have been used to explain paradoxical phenomena arising in cognitive science such as violations of rational decision

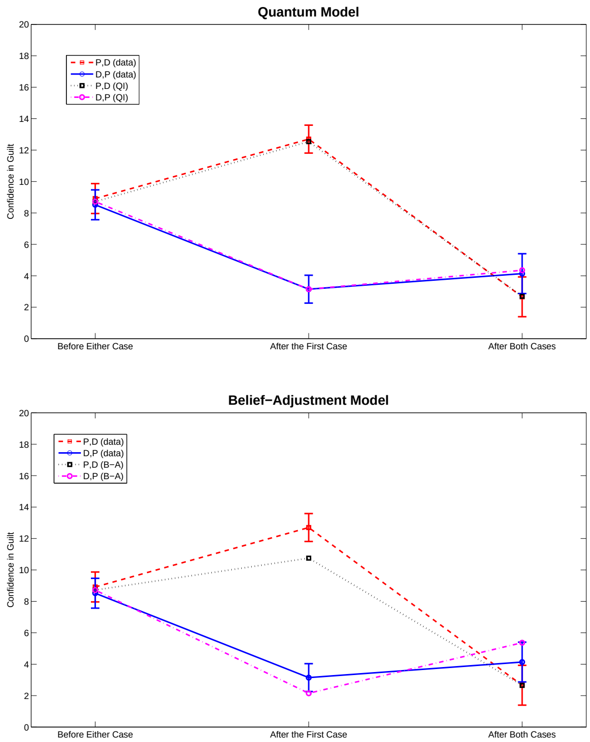

Figure 3: Adding and quantum model fits to the mean probability of guilt from experiment 2. Error bars on the data show the 95% confidence interval.

making principles (Pothos & Busemeyer, 2009), paradoxes of conceptual combination (Aerts, 2009), human judgments (Khrennikov, 2004), and perception (Atmanspacher, Filk, & Romer, 2004).

Here we provide evidence in support of a quantum probability explanation of order effects. Using data collected by McKenzie et al., we show that the quantum inference model out performs the minimum acceptable strength model. We also provide evidence that the quantum model performs as well or slightly better than the belief-adjustment model when fitting data from experiment 1. Finally, we describe some of the limitations of the belief-adjustment model in relationship to irrefutable evidence. We argue that the quantum inference model is not faced with these limitations and provides more reasonable predictions. In the future, we plan to continue empirically investigating the quantum inference model in the hope of developing a more coherent theory concerning human inference tasks.

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