Chapter 1

A Quantum Bayes Net Approach to Causal Reasoning

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When individuals have little knowledge about a causal system and must make causal inferences based on vague and imperfect information, their judgments often deviate from the normative prescription of classical probability. Previously, many researchers have dealt with violations of normative rules by elaborating causal Bayesian networks through the inclusion of hidden variables. While these models often provide good accounts of data, the addition of hidden variables is often post hoc, included when a Bayes net fails to capture data. Further, Bayes nets with multiple hidden variables are often difficult to test. Rather than elaborating a Bayes net with hidden variables, we generalize the probabilistic rules of these models. The basic idea is that any classic Bayes net can be generalized to a quantum Bayes net by replacing the probabilities in the classic model with probability amplitudes in the quantum model. We discuss several predictions of quantum Bayes nets for human causal reasoning.

1. Background and motivation

Human causal reasoning has intrigued scholars from a number of different fields including philosophy, cognitive science, and developmental psychology. Traditionally, models of causal reasoning have been based on classical probability theory. Early models were centered around the notion that people reason about causes and effects by observing the covariation between events [1, 2]. These models are rooted in ideas from Hume [3], who hy-
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pothesized that our sensory system infers causation through the constant conjunction of events. While covariational models can account for many phenomena, all models in this class have one distinct problem - the covariation of events does not necessarily imply a causal relationship between the events. For example, over the past century the number of pirates has decreased while average global temperatures have increased. While these two events negatively covary, no one would recommend becoming a pirate as a solution to global warming.

Cheng and Novick [4, 5] attempted to overcome these issues by incorporating domain specific prior knowledge with covariational information in their power PC theory. In their theory, people infer causal relationships to understand observed regularities in the occurrence of events. The model can explain why covariation sometimes implies causation and sometimes it does not. Despite the many successes of power PC theory, some studies have demonstrated that people’s judgments often deviate from the model’s predictions [6, 7].

Perhaps the most successful classical approach to modeling human causal reasoning is the one based on causal Bayesian networks (or causal Bayes nets for short). In a causal Bayes net, causal relationships are represented by Bayes’ calculus and model predictions are typically accepted as normative [8, 9]. This class of models has been shown to account for causal inferences driven by observation, intervention-based, and counterfactual processes [10], which are often difficult to discriminate with other classical approaches. Causal Bayes nets have also been used to explain how people learn causal relationships over time [11, 12]. Attempts have even been made to combine more traditional approaches, such as power PC theory, with causal Bayes nets [11, 13]. More generally, causal Bayes nets have been applied to classification [14, 15], decision making [16], and structured knowledge [17].

Despite their many successes in accounting for human causal reasoning, recent studies have suggested that people’s judgments sometimes deviate from the normative prescription of causal Bayes nets. In particular, people commonly violate a condition of causal Bayes nets called the local Markov property. This property is fundamental to causal Bayes nets and, in part, defines the probabilisitic rules of the net. The property states that if we have knowledge about all of the possible causes of some event $A$, then the descendants (i.e., the effects) of $A$ may tell us information about $A$, but the non-descendants (i.e., noneffects) do not. Recently, several studies have shown the people often violate the local Markov condition [18–23]. Relat-
edly, Fernbach et al. [24] found the people often ignore relevant variables in their causal judgments. In their study, people often ignored alternative causes in predictive reasoning situations (i.e., reasoning about an effect given the causes), but not in diagnostic reasoning situations (i.e., reasoning about the causes given the effects).

One way to overcome the issues mentioned above is to extend a causal Bayes net through the inclusion of hidden variables. These latent variables are considered part of the mental reconstruction of a causal system by an individual, but are never explicitly described in the experiment. Such approaches often provide good accounts of data [20]; however, they are difficult to conclusively test because the variables are never mentioned by the experimenter. Further, hidden variables are typically added to a causal Bayes net in a post hoc manner, after a basic causal Bayes net fails to explain behavioral phenomena. As an alternative approach, we propose generalizing causal Bayes net by altering the probabilistic rules that govern the net. The basic ideas is to replace the classical probabilities in a causal Bayes net with quantum ones, thereby producing quantum Bayes nets [25, 26]. In the next sections, we review causal Bayes nets in more detail and introduce quantum Bayes nets.

2. Causal Bayes nets

A causal Bayes net represents casual relationships between variables through a directed acyclic graph (DAG), which describes a set of random variables and their conditional dependencies. For example, suppose you are driving to work and run into a traffic jam. You might consider two possible causes for the traffic jam - (1) normal rush hour traffic or (2) an automobile wreck. In this example, the three variables - traffic jam, rush hour, and wreck - are each represented as nodes in the DAG (see Figure 1). The edges connecting the nodes represent conditional dependencies. In Figure 1, there is an edge between rush hour and traffic jam as well as wreck and traffic jam. If two nodes are not connected by an edge, then they are conditionally independent. In the example, there is no edge between wreck and rush hour because they are assumed to be conditionally independent.

The probability of a node having a specific value is determined by a probability function that uses the values of parent nodes as inputs. These probabilities are defined in conditional probability tables. Consider the traffic jam scenario where the three variables have two possible values: \( J = \) traffic jam is present (true/false), \( R = \) rush hour traffic is present.
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![Causal Bayes net of the traffic jam scenario.](image)

Fig. 1. A causal Bayes net of the traffic jam scenario. There are two possible causes of a traffic jam - rush hour traffic or a wreck. The three binary variables are represented as a DAG with conditional probability tables.

(true/false), and \( W = \) wreck is present (true/false). The causal Bayes net can address such questions as “What is the probability that there is a wreck, given that there is traffic jam?” by using the formula for conditional probability:

\[
p(W = t|J = t) = \frac{p(J = t, W = t)}{p(J = t)} = \frac{\sum_{j \in \{t, f\}} p(J = t, W = t, R = j)}{\sum_{i, j \in \{t, f\}} p(J = t, W = i, R = j)}
\]

(1)

where the joint probability function \( p(J = t, W = i, R = j) = p(J = t|W = i, R = j) p(W = i) p(R = j) \) because \( W \) and \( R \) are conditionally independent. The desired probability \( p(W = t|J = t) \) is then calculated using the conditional probability tables in Figure 1:

\[
p(W = t|J = t) = \frac{(0.9 \times 0.1 \times 0.7) + (0.8 \times 0.1 \times 0.3)}{(0.9 \times 0.1 \times 0.7) + (0.8 \times 0.1 \times 0.3) + (0.8 \times 0.9 \times 0.7) + (0.5 \times 0.9 \times 0.3)} \\
\approx 0.120
\]

(2)
The probabilities for other questions (e.g., \( p(W = f|J = f) \)) follow similar calculations.

A causal Bayes net obeys the local Markov property, which states that any node in a Bayes net is conditionally independent of its non-descendants (i.e., noneffects) given information about its parents (i.e., direct causes). Consider a causal situation where a variable \( A \) causes \( B \) and \( C \) (represented by the DAG: \( B \leftarrow A \rightarrow C \)). The local Markov property states that if you know \( A \), then \( B \) provides no new information about the value of \( C \). As a result, we have \( p(C|A) = p(C|A, B) \). Recently, researchers have found empirical evidence that people's causal judgments do not always obey the local Markov property. In particular, Rehder [20] gave participants causal scenarios with three binary variables (e.g., an economic situation with variables described as trade deficits, interest rates, and retirement savings) and asked them to judge the value of an unknown target variable given information about either or both of the remaining variables. Rehder's [20] results showed that information about non-descendants (i.e., noneffects) influenced judgments even when the values of direct causes were known, thus demonstrating a violation of the local Markov property. (We provide more details about this experiment in a section 4.1.)

Rehder [20] accounted for the observed violations of the local Markov property by elaborating a basic causal Bayes net with an additional variable that was either a shared mediator, shared disabler, or shared cause. For example, a common cause structure where \( A \) causes \( B \) and \( C \) can be augmented in several different ways by the inclusion of a fourth variable \( D \) as illustrated in Figure 2. Even though this approach can provide a reasonable account of human judgments, it is difficult to conclusively test because participants are never queried about the latent variable \( D \). As an alternative to using hidden variables, we propose generalizing causal Bayes nets to quantum Bayes nets.

3. Quantum Bayes nets

Following Tucci [25, 27] and Moreira et al. [28] our quantum Bayes nets are constructed by replacing the classical probabilities in the conditional probability tables of a causal Bayes net with quantum probability amplitudes. Consider a simple scenario where there are two causally related, binary (true/false) variables \( A \) and \( B \) such that \( A \rightarrow B \). In quantum probability
theory, these variables are represented by Hermitian operators:

\[ A = a_t P_{a_t} + a_f P_{a_f} \]  \hspace{1cm} (3a)

\[ B = b_t Q_{b_t} + b_f Q_{b_f} \]  \hspace{1cm} (3b)

where \( a_i \) and \( b_i \) are eigenvalues and \( P_{a_i} \) and \( Q_{b_i} \) are projectors onto the corresponding eigen-subspaces. The probability of a specific value, such as \( a_t \) (the notation \( a_t \) is shorthand for \( A = t \)), is given by Born’s rule:

\[ p(A = t) = \langle P_{a_t} \psi | \psi \rangle = ||P_{a_t} \psi||^2 \]  \hspace{1cm} (4)

where \( \psi \) is a unit length state vector representing an individual’s knowledge about the different variables.

Now, suppose we want to determine the probability that \( B \) is false given that \( A \) is true. To answer this question, we first calculate the conditional state vector \( \psi_{a_t} \) and then apply Born’s rule:

\[ p(B = f | A = t) = \langle Q_{b_f} \psi_{a_t} | \psi_{a_t} \rangle = ||Q_{b_f} \psi_{a_t}||^2. \]  \hspace{1cm} (5)

These quantum conditional probabilities are then used to define the conditional probability tables associated with the net.

Consider the traffic jam scenario again. With a causal Bayes net, we can answer questions such as “What is the probability that there is a wreck, given that there is a traffic jam?” by calculating joint probabilities such as \( p(J = t, W = i, R = j) \). These joint probabilities are determined from the conditional probability tables of the causal Bayes net by writing \( p(J = t, W = i, R = j) = p(J = t | W = i, R = j)p(W = i)p(R = j). \)

In our quantum Bayes net, we take a similar approach. First, let the three variables traffic jam, wreck, and rush hour be associated with projectors \( P \), \( Q \), and \( S \) respectively. We define joint probabilities by Born’s rule:

\[ p(J = t, W = i, R = j) = ||P_{j_t} \psi_{w_i, r_j}||^2 ||Q_{w_i} \psi||^2 ||S_{r_j} \psi||^2. \]  \hspace{1cm} (6)
where the conditional state is given by
\[
\psi_{w_i,r_j} = \frac{S_{r_j}Q_{w_i}\psi}{||S_{r_j}Q_{w_i}\psi||},
\]
(7)
If the projectors \(Q\) and \(S\) do not commute, then the conditional state will depend on the order in which these two variables are considered so that \(\psi_{w_i,r_j} \neq \psi_{r_j,w_i}\). As a result, \(p(J = t|W = i, R = j) \neq p(J = t|R = j, W = i)\).

Figure 3 shows a quantum Bayes net generalization of the traffic jam causal Bayes net shown in Figure 1. The probabilities in the causal Bayes net have been replaced by probability amplitudes in the quantum Bayes net. These amplitudes can be related to classical probabilities by squaring the magnitude of the amplitudes. For example, the probability that there was a wreck is given by
\[
p(W = t) = ||.3162 e^{i\theta_{wt}}||^2 = (.3162 e^{i\theta_{wt}})(.3162 e^{-i\theta_{wt}})
= (.3162)^2 e^{i(\theta_{wt}-\theta_{wt})} = .1
\]
(8)
which is identical to the classical probability in the causal Bayes net. Note that the term \(e^{i\theta_{wt}}\) is simply the phase of the amplitude.

When calculating the conditional probabilities of the traffic jam given information about a wreck and rush hour, the order in which the variables wreck and rush hour are considered is important. In the quantum Bayes net shown in Figure 3, we assume that wrecks are always considered before rush hour. Psychologically, this means that an individual thinks about wrecks and rush hour separately, always starting with wrecks. This is represented in the figure by the thick border on the wreck node. If we want to switch the order and have rush hour processed before wrecks, then we would need to define a different set of conditional probabilities. In other words, there are two different conditional probability tables describing the probability of the traffic jam given information about wrecks and rush hour - one table describing the scenario where wrecks is processed before rush hour (shown in Figure 3) and another table describing the scenario where rush hour is considered before wrecks (not shown). The two conditional probability tables are related to one another in a specific manner. In quantum probability theory, noncommutative events (such as wreck and rush hour) are related by a unitary transformation, which preserves lengths and inner products.
For the traffic jam scenario, we started with a causal Bayes net and generalized this to a quantum Bayes net by designating a processing order (wreck before rush hour) and replacing classical probabilities by probability amplitudes. Note that the decision that wrecks should be processed before rush hour was arbitrary. We could have easily chosen the reverse order (rush hour before wrecks). That is, there are at least two different ways to generalize the causal Bayes net in this example. In general, there will often be multiple ways to turn a causal Bayes net into a quantum one. That is, if you start with a well-defined causal Bayes net, there are several ways to convert it into a quantum one because of the arbitrariness in the order of events. This is because noncommutative events result in different conditional probability tables for the same causal scenario. Thus, the behavior of a quantum Bayes net will often be fundamentally different than that of a causal Bayes net.

Fig. 3. A quantum Bayes net generalization of the traffic jam scenario. The wreck node in the DAG has a thick border to indicate that it is considered before rush hour. The tables contain probability amplitudes rather than probabilities. These amplitudes were determined from the causal Bayes net shown in Figure 1. The term $e^{i\theta_n}$ is the phase of the amplitude associated with situation $n$. For example, the probability amplitude for $J = t$ given that $W = t$ and $R = t$ (first row of the bottom conditional probability table) has a phase of $e^{i\theta_n}$ where the index is $n = jt|wt,rt$. 

<table>
<thead>
<tr>
<th>Rush hour</th>
<th>Wreck</th>
<th>Traffic jam</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>t</td>
<td>$0.3162e^{i\theta_{wt}}$</td>
</tr>
<tr>
<td>f</td>
<td>f</td>
<td>$0.7071e^{i\theta_{rf}}$</td>
</tr>
</tbody>
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<table>
<thead>
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<th>Rush hour</th>
<th>Wreck</th>
<th>Traffic jam</th>
</tr>
</thead>
<tbody>
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<td>t</td>
<td>$0.9487e^{i\theta_{wf}}$</td>
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<td>f</td>
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<table>
<thead>
<tr>
<th>Rush hour</th>
<th>Wreck</th>
<th>Traffic jam</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>f</td>
<td>$0.4472e^{i\theta_{rf}}$</td>
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<td>f</td>
<td>f</td>
<td>$0.7071e^{i\theta_{rf}}$</td>
</tr>
</tbody>
</table>
4. Predictions from quantum Bayes nets

Quantum Bayes nets with noncommutative events generate several interesting predictions regarding human causal judgments. In the following sections, we discuss these predictions and the supporting empirical evidence.

4.1. Violations of the local Markov condition

The local Markov property states that any node in a DAG is conditionally independent of its nondescendants when its parents (direct causes) are known. For example, in the common effect structure $A \rightarrow C \leftarrow B$, the two causes $A$ and $B$ are conditionally independent. That is, if $A$ and $B$ are binary variables, then $p(A = i | B = t) = p(A = i | B = f)$ for $i \in \{t, f\}$ and likewise when $A$ and $B$ are swapped. In a quantum Bayes net, if $A$ and $B$ do not commute, then there is a dependency between these two variables such that knowing information about $B$ influences our beliefs about $A$. This dependency leads to violations of the local Markov condition because it implies $p(A = i | B = t) \neq p(A = i | B = f)$. By the definition of conditional probability, $p(A = i | B = j) = p(A = i, B = j) / p(B = j)$. In a causal Bayes net, we have $p(A = i, B = j) = p(A = i) p(B = j)$ because $A$ and $B$ are independent. As a result, $p(A = i | B = j) = p(A = i)$ for all $i, j$. In a quantum Bayes net, $p(A = i, B = j) = ||P_a Q_b \psi||^2$, and if $A$ and $B$ do not commute, then clearly $||P_a Q_b \psi||^2 \neq ||P_a \psi||^2 ||Q_b \psi||^2$, leading to violations of the local Markov condition.

Rehder [20] showed that people’s causal judgments often violate the local Markov condition. In his experiment, participants read information about two causal scenarios involving an unknown target variable and were asked to choose the scenario where the target variable was more probable. For example, in the common effect situation where $A \rightarrow C \leftarrow B$, participants had to choose whether the target variable $B$ was more likely to be true in a scenario where $A = t$ or in a scenario where $A = f$. A causal Bayes net predicts that participants’ choice proportions should be equal on average because $p(B = t | A = t) = p(B = t | A = f)$ by the local Markov property. However, Rehder [20] found that people tended to select the causal scenario where $A = t$ more often than the alternative where $A = f$. These results suggest that people judged $p(B = t | A = t) > p(B = t | A = f)$. Rehder found similar violations of the local Markov property with other causal structures such as common cause structures ($B \leftarrow A \rightarrow C$) and chain structures ($A \rightarrow B \rightarrow C$).
4.2. Anti-discounting behavior

Quantum Bayes nets with noncommutative events can also explain anti-discounting behavior in causal reasoning. The term discounting is used to describe a situation where one cause casts doubts on another cause. For example, in the common effect structure $A \rightarrow C \leftarrow B$, knowledge of $A$ could cast doubt on the value of $B$ such that $p(B|C, A) < p(B|C)$. In many causal situations discounting is normative [29]. To see why this might be the case, consider the conditional probabilities $p(B = 1 | C = 1, A = 1)$ and $p(B = 1 | C = 1)$. It is normatively correct to judge $p(B = 1 | C = 1) > p(B = 1 | C = 1, A = 1)$ because knowing $A = 1$ provides sufficient explanation for the value of the effect $C = 1$, thus making the other cause $B$ redundant. When the value of $A$ is unknown, there is a greater chance the effect $C$ was brought about by $B$.

Rehder [20] showed that many people display anti-discounting behavior. In his experiments, people judged an unknown target variable $B$ as highly likely when an alternative cause $A = 1$ was known, leading to judgments where $p(B = 1 | C = 1, A = 1) < p(B = 1 | C = 1)$. Similar to violations of the local Markov property, quantum Bayes nets explain anti-discounting behavior by the noncommutativity of $A$ and $B$. When two events do not commute, there is a causal dependency between them. Thus knowing the value of one cause can increase the probability of another cause having a similar value, resulting in judgments that are opposite to those predicted by discounting.

4.3. Order effects

If two events are noncommutative, then quantum Bayes nets naturally predict order effects. In an experiment involving the common effect scenario $A \rightarrow C \leftarrow B$, participants might be asked to judge $p(C|A, B)$ where they are given information about $A$ before $B$. An order effects occurs when an individual’s final judgment depends on the order in which the information was presented, so that $p(C|A, B) \neq p(C|B, A)$. Causal Bayes nets have difficulty accounting for order effects because they are based on classical probability theory and thus obey the commutative property. That is, $p(A, B|C) = p(B, A|C)$ implies $p(C|A, B) = p(C|B, A)$ by Bayes’ rule. Classical models can be extended to account for order effects, but this involves introducing extra variables such as $O_1$, the event that $A$ is presented first, and $O_2$, the event that $A$ is presented second. With these additional events, then it is possible to have $p(C|A, B, O_1) \neq p(C|A, B, O_2)$. However,
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without a theory about $O_1$ and $O_2$, this model simply redescribes the empirical result. Moreover, in most studies of order effects, the presentation order is randomly determined, yielding order information such as $O_1$ and $O_2$ irrelevant.

Many empirical studies have documented the importance of order information in human judgments [30]. Order effects have been observed in a number of different domains ranging from judging the probability of selecting balls from urns [31] to judging the guilt of a defendant in a mock trial [32, 33]. Trueblood and Busemeyer [34] demonstrated that order effects also arise in causal reasoning. In their experiment, participants were asked to make judgments about ten different causal scenarios involving a single effect and two binary (present / absent) causes. For example, in one scenario, participants read a description about a college sophomore, Liz, with a 3.0 GPA and were asked to judge the likelihood that Liz will earn an A in her social psychology class. In one condition, participants first read information about how Liz hopes to study social work in graduate school (the present cause) followed by information that Liz does not make any changes to her study habits (the absent cause). In a different condition, participants read the information in the reverse order.

The participants (N = 113) provided likelihood judgments of the effect (e.g., Liz receiving an A in social psychology) on a 0 to 100 scale at three different time points: (1) before reading either cause, (2) after reading one of the causes, and (3) after reading the other cause. Participants judged the present cause before the absent cause in a random half of the scenarios. In the other half of the scenarios, the order of the causes was reversed. The results of the experiment showed a large, significant order effect ($p < 0.001$) across all ten scenarios.

5. Conclusions

Arguably, causal Bayes nets have been one of the most successful modeling approaches to human causal reasoning. These models can account for a range of findings including causal deductive and inductive reasoning across a number of different scenarios. Further, causal Bayes nets have been applied to other domains such as decision-making [16] and classification [14, 15, 35].

Despite their many successes, recent experiments have shown that people's judgments often deviate from the predictions of causal Bayes nets. In particular, people often violate the local Markov property of causal Bayes nets. To overcome such issues, researchers have suggested augmenting
causal Bayes nets with additional variables [20]. These hidden variables provide increased model flexibility, allowing causal Bayes nets to accommodate a wider range of behavior. However, the addition of such variables is often post hoc and they are difficult to conclusively test experimentally.

As an alternative, we suggest generalizing causal Bayes nets using quantum probability theory. In our approach, we do not elaborate a causal Bayes net with additional nodes and edges. Rather, we change the probabilistic rules used to perform inference. By using quantum probabilities instead of classical ones, we can allow for variables to be noncommutative. By using noncommutative events in our quantum Bayes nets, we can account for a number of different behavioral phenomena including violations of the local Markov condition, anti-discounting behavior, and order effects.

Quantum probability theory has been successfully applied to a range of different problems in the cognitive and decision sciences including interference effects in perception [36], violations of the sure thing principle [37], violations of dynamic consistency [38], conjunction and disjunction fallacies [39], order effects in survey questions [40], and the interference of categorization on decision-making [41]. We believe that quantum probability theory also has great potential to explain human causal reasoning. The results discussed in this chapter make us optimistic about this approach in the future.

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