Quantum Model for Conjoint Recognition

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Abstract

In a conjoint memory recognition task, a person is presented a list of target items to remember. Afterwards, a test probe is presented which is sampled from one of three mutually exclusive and exhaustive categories: one is a target from the set of previously presented targets; a second is a non target but meaningfully related to a target; and a third is a non target and unrelated to any target. The episodic overestimate effect refers to the fact that the probability of accepting a probe when asked if it is a target plus the probability of accepting a probe when asked if it is a related non target is greater than the probability of accepting a probe when asked if it is either a target *or* a non related target. Logically these two probabilities should be identical. Previously these results were explained by a dual process theory. This article presents an alternative quantum memory recognition model for this effect that addresses some problematic issues that arise with the dual process explanation.

Quantum Models of Cognition

Beginning with seminal ideas by Aerts and Aerts (1994), Atmanspacher et al. (2002) and Khrennikov (1999), a growing number of scientists have begun to explore the use of quantum formalisms to understand what appear to be paradoxical findings in various areas of human cognition and decision making. A variety of quantum theoretical applications have appeared including applications to conceptual judgments, human inference, decision making behavior, and human memory (see Bruza, Busemeyer, & Gabora, 2009).

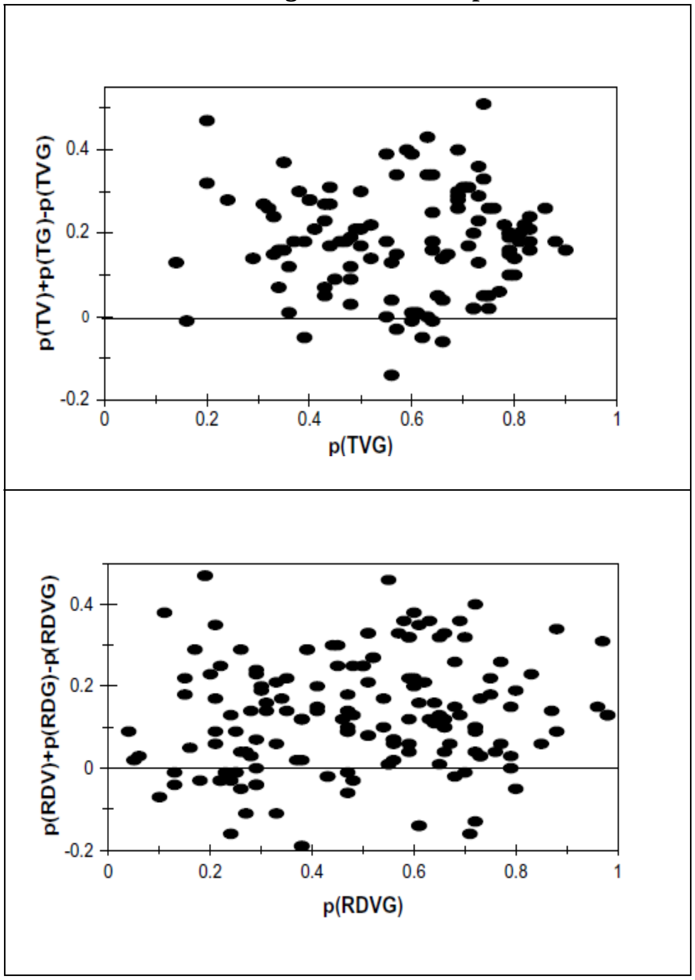
The purpose of this paper is to present a new application of quantum theory to a puzzling phenomenon observed in human memory recognition called the episodic over-distribution effect, which was discovered by Brainerd and Reyna (2008). First we describe the phenomena and explain why it is a puzzle, and then we present a quantum solution to the puzzle.

The Episodic over-distribution Effect

The phenomenon of interest is observed in human memory experiments that use a memory recognition paradigm called the conjoint – recognition paradigm. Initially, participants are rehearsed on a set T of memory targets (e.g., each member is a short description of an event). After a delay, a recognition test phase occurs, during which they are presented a series of test probes that consist of trained targets from T, related non-targets from a different set R of events (e.g. each member is a new event that has some meaningful relation to a target event), and unrelated set U of non-target items (e.g. each member is completely unrelated to the targets). During the memory test phase, three different types of recognition instructions are employed: the first is a verbatim instruction (V) that requires one to accept only exact targets from T; the second is a gist instruction (G) that requires one to accept only related non targets from R; the third is an instruction to accept verbatim or gist items, that is it requires one to accept probes from either from T or R. Note that $T \cap R = \emptyset$, and so logically V and G are supposed to be mutually exclusive alternatives.

First consider memory test trials that employ a test probe belonging to the target set T. If the verbatim question is asked, then the probability of accepting the target is denoted P(TV); if the gist question is asked, then the probability of accepting the target is denoted P(TG); and if the verbatim or gist question is asked the probability of accepting the target is denoted P(TVG). Logically, a probe x comes from T or G but not both, implying that P(TVG) = P(TV) + P(TG). The difference, EOD(T) = P(TV)+P(TG) – P(TVG) is an episodic over distribution effect. Figure 1a shows the observed EOD effects obtained from 116 different experimental conditions reported by Brainerd and Reyna (2008). All but 12 (10%) of the 116 studies produced this effect, and the mean value of the EOD equals .18 (after removing the bias effect described below).

Figure 1a and 1b. Episodic over distribution (EOD) effects from trained targets and related probes.



Next consider the memory test trials that employ a test probe belonging to the related set R. If the verbatim question is asked, then the probability of accepting the probe is denoted P(RV); if the gist question is asked, then the probability of accepting the probe is denoted P(RG); and if the verbatim or gist question is asked, then the probability of accepting the probe is denoted P(RVG). Once again, the test probe came from T or G but not both, implying that P(RVG) = P(RV) + P(RG). The difference, EOD(V) = P(RV)+P(RG) − P(RVG) is another episodic over distribution effect. Figure 1b shows the observed EOD effects obtained from 165 different experimental conditions reported by Brainerd and Reyna (2008). All but 17 of 165 studies produced the effect. The mean value of the EOD is .13 (after removing the bias effect described below), which is smaller than that obtained with the target probe.

Finally consider memory test trials that employ a test probe belonging to the unrelated non target set U. If the verbatim question is asked, then the probability of accepting the probe is denoted P(UV); if the gist question is asked, then the probability of accepting the probe is denoted P(UG); and if the verbatim or gist question is asked, then the probability of accepting the probe is denoted P(UVG). Brainerd and Reyna (2008) didn't report these probabilities, but Brainerd et al. (1999) reported statistical tests which indicated that P(UV) < P(UG) < P(UVG). Furthermore, four out of the six studies exhibited episodic over distribution (EOD) effects, and the mean across the six studies reported in Table 6 of Brainerd et al. (1999) shows that EOD(U) = P(UV)+P(UG) − P(UVG) = .04 (which is comparable to EOD(T) = .05 found with the target probes in these six studies). The EOD effect in this case is even smaller than that for the related non target probe, but it is still present.

Another important finding was that the probabilities for the verbatim and gist instructions often summed to a value greater than unity. The sum P(TV)+P(TG) exceeded one for 20 of the 116 (17%) data sets involving true targets, and the sum P(RV)+P(RG) exceeded one for 12 of the 165 (7%) related non targets.

1. Memory Strength Model

Not only are these EOD effects inconsistent with a logical analysis of the task, they are also inconsistent with a memory strength model of recognition memory (e.g. Wixted, 2007). According to the memory strength model, each test probe produces some feeling of familiarity that has a strength value denoted by a random variable f. If the strength of the familiarity is below a cutoff c_R, then it is categorized as an unrelated probe; if it falls above c_R but falls below a higher cutoff c_T then it is categorized as a related non target; and if it falls above the upper cutoff c_T, then it is categorized as a true target. This model satisfies the mutually exclusive principle because the strength must fall into one of three intervals ($f < c_R$, $c_R \leq f \leq c_T$, $f > c_T$) and so it cannot produce episodic over estimation either. For a probe of type $x \in \{T,R,U\}$, this model predicts that

$$P(xV) = \Pr[\, f_x > c_T], \quad (1)$$
$$P(xG) = \Pr[\, c_R \leq f_x \leq c_T \,] \text{ and}$$
$$P(xVG) = \Pr[\, f_x \geq c_R] = \Pr[\, c_R \leq f_x \leq c_T \,] + \Pr[\, f_x > c_T].$$

The latter always predicts that EOD(x) = 0.

2. Classic Probability Model

It seems reasonable to interpret the probabilities elicited by the 'verbatim or gist' instruction as the probability of the union of the two events V = 'categorize as verbatim', G = 'categorize as gist.' It also seems reasonable to allow the possibility that the participants failed to follow the mutually exclusive instructions, so that they were willing to implicitly categorize a probe as either verbatim or gist item or *both*. This implies the classic probability model

$$P(V \cup G|x) = P(V|x) + P(G|x) - P(V \cap G|x) \quad (2)$$
$$< P(V|x) + P(G|x).$$

Under this assumption, episodic over distribution provides a measure of the *joint* probability that a probe is categorized as both verbatim and gist:

$$P(V \cap G|x) = P(V|x) + P(G|x) - P(V \cup G|x) = EOD(x).$$

This model also allows $P(G|x)+P(V|x) > 1$, which is occasionally found.

This simple explanation can be empirically tested by obtaining a second estimate of the joint probability from the data. The participants were informed that the verbatim and gist properties were mutually exclusive, $P(V|G, x) = 0 < P(V|x)$, and so it is safe to assume that $P(G \cap V|x) = P(G|x) \cdot P(V|G, x) < P(G|x) \cdot P(V|x)$ which is the bound produced by assuming statistical independence. Given the instructions, the joint probability should fall far below the independence bound. Therefore, if this simple explanation is correct, then the EOD effect should certainly be bounded by the product of the individual probabilities, $EOD(x) < P(G|x) \cdot P(V|x)$.

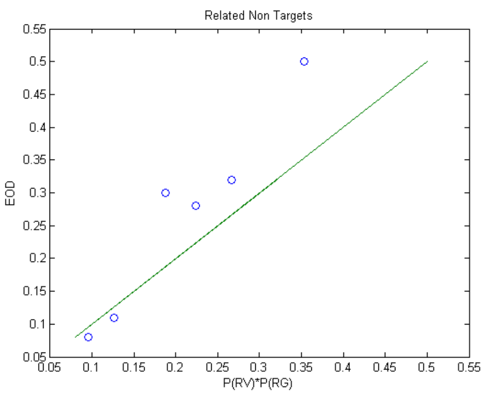


Figure 2: EOD plotted as a function of product of probabilities for related non targets presented in Brainerd et al. (1999) Table 2

Figure 2 shows that this independence bound is systematically violated. The data in Figure 2 are from Table 2 of Brainerd et al. (1999) for the related non targets, which happen to produce the largest EOD effects in this set of six studies. All points lying above the unit line are violations of the independence bound. Large violations tend to occur when the product is large.

A more striking violation of the classic probability model can be seen in Figure 1. If the EOD effect equals the joint probability that a probe x is verbatim *and* gist, and $P(xVG)$ equals the disjunctive probability that the probe x is verbatim *or* gist, then according to classic probability theory, the probability of the intersection (i.e. EOD) must be less than the probability of the union (i.e. $P(xVG)$). Figure 1a shows that when $P(TVG) < .20$, the EOD effect for targets in T sometimes exceeds $P(TVG)$. This finding also appears again in Figure 1b, which shows that the EOD effect for related non targets in R often exceeds $P(RVG)$ for $P(RVG) < .20$. Therefore, contrary to the assumptions of a classic probability model, the EOD effect exceeds $P(xVG)$ when the latter is small.

3. Dual Process Model.

Jacoby (1991) proposed a dual process model of memory recognition which was later extended by Brainerd et al. (1999), and the latter is presented here. This is a Markov model that posits three states. First we focus on test probes that are true targets from T. In this case, the person may correctly recollect the target (denoted by state *S*) with probability p_S, or fail to recollect but instead enter a familiar state *F* with probability p_F, or the person may not recollect and not be familiar. According to this model, the probabilities of responses to questions for a probe that is a true target item from T are given by

$$\begin{aligned} P(TV) &= p_S + (1 - p_S) \cdot p_F \\ P(TG) &= (1 - p_S) \cdot p_F \\ P(TVG) &= p_S + (1 - p_S) \cdot p_F \end{aligned} \qquad (3a)$$

The first equation assumes that either the person will recollect the true target from T or not recollect it but consider it sufficiently familiar to accept (although the latter is inconsistent with the instruction). The second equation assumes the person does not recollect the item but considers it sufficiently similar to accept it as a non target but related item. The third equation is interpreted in the same way as the first.

Obviously, the dual process model predicts episodic over extension because

$$EOD(T) = P(TV)+P(TG) - P(TVG) = (1-p_S) \cdot p_F.$$

It can also produce sums of $P(TV)+P(TG)$ that exceed unity (e.g., set $p_S = .10$ and $p_F = .90$).

If we interpret $P(TVG)$ as the disjunctive probability $P(V \cup G|T)$ of categorizing the target as verbatim or gist, then the dual process model *violates* classic probability theory. This is because according to the dual process model $P(TVG) = P(TV)$, but classic probability theory requires $P(V \cup G|x) = P(V|x) + P(\sim V \cap G|x) > P(V|x)$. Thus the dual process model does not have to obey the independence bound $EOD(x) < P(G|x) \cdot P(V|x)$. However, the dual process model incorrectly predicts that $P(TVG) = P(TV)$ when in fact $P(TVG) > P(TV)$ (see Table 2, Brainerd et al., 1999).

To overcome the problem mentioned above, Buchner et al. (1998) added bias parameters which are estimated from the trials when the probe is taken from the unrelated set U. The bias parameter for the verbatim instruction equals $\beta_V = P(UV)$, the bias for the gist instruction equals $\beta_G = P(UG)$, and the bias for the V or G instruction equals $\beta_{VG} = P(UVG)$.

These biases are added to Equation 1 and they apply to the case when the probe is not recollected and is unfamiliar:

$$P(TV) = p_S + (1-p_S)\cdot p_F + (1-p_S)\cdot(1-p_F)\cdot\beta_V \qquad (3b)$$
$$P(TG) = (1-p_S)\cdot p_F + (1-p_S)\cdot(1-p_F)\cdot\beta_G$$
$$P(TVG) = p_S + (1-p_S)\cdot p_F + (1-p_S)\cdot(1-p_F)\cdot\beta_{VG}.$$

The bias corrected probabilities are then defined by

$$P'(TV) = P(TV) - (1-p_S)\cdot(1-p_F)\cdot\beta_V,$$
$$P'(TG) = P(TG) - (1-p_S)\cdot(1-p_F)\cdot\beta_G, \text{ and}$$
$$P'(TVG) = P(TVG) - (1-p_S)\cdot(1-p_F)\cdot\beta_{VG}.$$

This extended model predicts that

$$P'(TV) = P'(TVG) \text{ and}$$
$$EOD'(T) = P'(TG)+P'(TV)-P'(TVG)$$
$$= (1-p_S)\cdot p_F = P'(TG).$$

Brainerd and Reyna (1998) tested this prediction by regressing the bias corrected EOD on the estimate of $(1-p_S)\cdot p_F$. This produced a regression coefficient equal to .81 (R^2 = .36), which was not significantly different from the predicted coefficient equal to unity. It is important to note, however, that some studies produced large EOD' effects even when $(1-p_S)\cdot p_F$ was estimated to be zero (see Figure 5 of Brainerd & Reyna, 2008).

In an analogous manner, the dual process model posits that a related non target probe may generate one of three states in the person – a state S in which the probe is correctly categorized as a related non target with probability q_S, or fail to correctly categorize the probe as a related probe but instead enter a familiar state F with probability q_F, or the person may not correctly categorize the probe as a related non target and not be familiar with it either. The probabilities to questions for a probe that is a related non target are given by

$$P(RG) = q_S + (1-q_S)\cdot q_F + (1-q_S)\cdot(1-q_F)\cdot\beta_V \qquad (3c)$$
$$P(RV) = (1-q_S)\cdot q_F + (1-q_S)\cdot(1-q_F)\cdot\beta_G$$
$$P(RVG) = q_S + (1-q_S)\cdot q_F + (1-q_S)\cdot(1-q_F)\cdot\beta_{VG}.$$

The same acceptance rates for non target and unrelated probes taken from set U are again used to estimate the extra bias terms.

Although the dual process model provides a good description of the findings from the conjoint memory recognition paradigm, several important issues remain unresolved. First, the model provides no explanation for the EOD effect that remains even for test probes taken from the unrelated non target set U. In this case, there is no possibility for recollection or familiarity, and some unknown bias must be evoked to explain the EOD effect. Second, large EOD effects occur in some studies in which the model predicts zero effects. The latter is a dramatic failure of the model.

4. Quantum Recognition Model.

The episodic over distribution effect can be interpreted as a type of interference effect that naturally arises in quantum probability systems (Khrennikov, 2010). Here we apply a quantum model that was previously developed to explain human probability judgment errors (Busemeyer, Pothos, Franco, 2009). Other quantum models of human memory have been also been proposed by Buza et al. (2009) and Franco (2009).

The *first* postulate of the quantum model is that a person's memory is represented by a state vector, denoted $|\psi\rangle$, within a large but finite dimensional vector space H. Each dimension (basis vector) of the vector space represents a feature pattern that could be used to describe a past episode or event. A feature pattern is interpreted as a combination of many individual features. For example, experiencing the word 'poodle' on 'list 1' during rehearsal would represent a simple combination of two features (poodle, list 1). More complex combinations can be used to form a feature pattern, and so the dimensionality of the vector space can be large, but the finite dimension of the vector space puts limits on the capacity for describing past events or episodes. The use of feature vectors to represent memory is consistent with other memory recognition models (e.g., Hintzman, 1988; Shiffrin & Steyvers, 1997).

The state vector $|\psi\rangle$ represents beliefs about past events or episodes. Each coordinate value assigned by $|\psi\rangle$ to a particular dimension (basis vector) is called the probability amplitude (a complex number) for that feature pattern. The squared magnitude of the amplitude equals the probability that a feature pattern describing an event is considered to be true by the person. The state vector $|\psi\rangle$ has unit length so that the sum of the squared magnitudes equals one.

The *second* postulate is that an answer to a question about a test probe is represented by a subspace S of the vector space H. Intuitively, a subspace is described by a subset of the complete set of feature patterns that are used to describe the entire vector space. Technically, a subspace is spanned by a subset of a complete set of orthogonal basis vectors that span the entire vector space.

When the person is given the verbatim instruction, the answer yes to the probe from set T is represented by a subspace denoted S_{TV}, and the answer no is represented by the orthogonal complement $S_{TV}^{\perp}$. Alternatively, when the person is given the gist instruction, the answer yes to a probe from set T is represented by a different subspace S_{TG}, and the orthogonal complement $S_{TG}^{\perp}$ is used for no. When given a test probe from the related non target set R under the gist instruction, then the subspace for yes is S_{RG} and the subspace for no is $S_{RG}^{\perp}$. When given a test probe from the unrelated non target set U under the verbatim instruction, then the subspace for yes is denoted S_{UV} and the subspace for no is $S_{UV}^{\perp}$. Subspaces for the remaining questions are denoted using a similar notation.

Corresponding to each subspace S is a projector, denoted M_S, which is an idempotent linear operator ($M_S \cdot M_S = M_S$) that maps an arbitrary state vector $|\psi\rangle$ onto the subspace S to produce the projection $M_S\cdot|\psi\rangle$. Intuitively, the projector M_S serves as a retrieval cue, and the projection $M_S\cdot|\psi\rangle$ is the retrieved image. The projector for the no answer to this question is the projector for the orthogonal complement, $(I-M_S)$, where I is the identity operator.

Our *third* postulate is that the probability of answering yes to a question equals the squared length of the orthogonal projection of the state $|\psi\rangle$ onto the subspace S representing the answer yes to the question. Formally, the probability of answering yes equals $|M_S\cdot|\psi\rangle|^2$. Note that this probability can also be expressed as an inner product

$$|M_S\cdot|\psi\rangle|^2 = \langle\psi|M_S\cdot M_S|\psi\rangle = \langle\psi|M_S|\psi\rangle.$$

For example, the probability of saying yes to a probe from set $x \in \{T,R,U\}$ under the verbatim instruction equals $|M_{xV}\cdot|\psi\rangle|^2$, whereas the probability of yes for the same probe under the gist instruction equals $|M_{xG}\cdot|\psi\rangle|^2$. The probability of saying no to a probe from set x under the verbatim instruction equals $|(I - M_{xV})\cdot|\psi\rangle|^2 = 1-|M_{xV}\cdot|\psi\rangle|^2$.

Our *fourth* postulate concerns the revision of the state following an answer to a question. Suppose the person concludes that the answer to the gist question about a probe from set $x \in \{T,R,U\}$ is true. Then the initial state $|\psi\rangle$ undergoes a revision into a new state

$$|\psi_{xG}\rangle = M_{xG}\cdot|\psi\rangle/|M_{xG}\cdot|\psi\rangle|,$$

which is the normalized projection onto the yes answer to the gist question. If a later question is asked about the verbatim property for this same probe, then the probability of saying yes to the verbatim question (conditioned on saying yes to the gist question) equals $|M_{xV}\cdot|\psi_{xG}\rangle|^2$.

On the basis of the fourth postulate we can derive formulas for the probabilities of sequences of events. If the gist question is asked first and the verbatim question is asked second, then the probability of saying yes to the gist question *and* then saying yes to the verbatim question equals

$$|M_{xG}\cdot|\psi\rangle|^2\cdot|M_{xV}\cdot|\psi_{xG}\rangle|^2 = |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2. \quad (4a)$$

If these two questions are mutually exclusive, then $M_{xV}\cdot M_{xG} = M_{xG}\cdot M_{xV} = 0$.

If the gist question is asked first and the verbatim question is asked second, then the probability of answering yes to gist *or* then saying yes to the verbatim then equals

$$P(xVG) = |M_{xG}\cdot|\psi\rangle|^2 + |M_{xV}\cdot(I-M_{xG})|\psi\rangle|^2. \quad (4b)$$

We can expand the second term in Equation 4b into a form that is more useful for later comparisons:

$$|M_{xV}\cdot(I-M_{xG})|\psi\rangle|^2 = |M_{xV}\cdot|\psi\rangle|^2 + |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2 - \langle\psi|M_{xG}\cdot M_{xV}|\psi\rangle - \langle\psi|M_{xV}\cdot M_{xG}|\psi\rangle. \quad (4c)$$

Our *fifth* postulate concerns the commutative relations between the projectors for the verbatim and gist instructions. In general, the feature patterns used to describe the verbatim instruction are different than those used to describe the gist instruction. The verbatim instruction may orient the person towards the use of detailed surface features such as perceptual features, while the gist instruction may orient the person towards the use of deeper semantic features. Technically, the set of orthogonal basis vectors used to define the verbatim subspace is different than the set of orthogonal basis vectors used to define the gist subspace.

There are three fundamentally different ways to represent the relations between the projectors. The first way is to assume that the subspaces are orthogonal. Intuitively, the feature patterns used to describe the verbatim and gist instructions are mutually exclusive. Technically this means that all the basis vectors used to describe the verbatim instruction are orthogonal to all the basis vectors used to describe the gist instruction. More succinctly, this implies $M_{xV}\cdot M_{xG} = 0$. This is in fact how the sets of probes, T and R, were formed. However, this representation predicts no EOD effects as in Equation 1.

The second way is to assume that the projectors are compatible. Intuitively, this means that a common set of feature patterns are used to describe both the verbatim and gist instructions. In this case, the order that questions are asked does not make any difference. Technically, the projectors are compatible if $M_{xV}\cdot M_{xG} = M_{xG}\cdot M_{xV}$. However, if the projectors are compatible, then

$$\begin{aligned}\langle\psi|M_{xG}\cdot M_{xV}|\psi\rangle &= \langle\psi|M_{xV}\cdot M_{xG}|\psi\rangle \\ &= \langle\psi|M_{xV}\cdot M_{xG}\cdot M_{xG}|\psi\rangle = \langle\psi|M_{xG}\cdot M_{xV}\cdot M_{xG}|\psi\rangle \\ &= |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2\end{aligned}$$

and Equation 4b reduces to the same form as the classic probability model:

$$P(xVG) = |M_{xG}\cdot|\psi\rangle|^2 + |M_{xV}\cdot|\psi\rangle|^2 - |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2.$$

Given the instructions stating that the verbal and gist probes are mutually exclusive, we expect that $|M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2 < |M_{xV}|\psi\rangle|^2\cdot|M_{xG}|\psi\rangle|^2$. Therefore this assumption once again predicts that the EOD effect is bounded by the product of individual probabilities. But recall that this bound is violated as shown in Figure 2 of this paper.

The third way is to assume that the projectors are incompatible. Intuitively, people do not use a common set of feature patterns to describe verbatim and gist instructions, but these features are not mutually exclusive either. In this case, the subspace for the verbatim instruction does not share the same basis vectors as the subspace for the gist instruction, but they are not orthogonal either. This implies that $M_{xV}\cdot M_{xG} \neq 0$, and moreover, $M_{xV}\cdot M_{xG} \neq M_{xG}\cdot M_{xV}$, so that these two projectors do not commute. Thus the order of application does matter, which is consistent with the finding that the order that information is retrieved from memory is important and can affect the final probabilities of answers (Storm, Bjork, & Bjork, 2007). In particular, for the conjoint recognition paradigm, the quantum model predicts that the probability of saying yes to the verbatim question followed by yes to the gist question equals $|M_{xG}\cdot M_{xV}\cdot|\psi\rangle|^2$, but the probability of the reverse order equals $|M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2$. These two probability sequences will not necessarily be equal because the two projectors do not commute in general. As we will later show, according to quantum theory, this is the key reason for the episodic overestimation effect.

Quantum Explanation for the Episodic Over Distribution Effect.

The key to understanding the episodic overestimation effect lies in the theoretical analysis of the 'verbatim or gist' instruction. According to the quantum model, this disjunctive question entails the use of two projectors. If the projectors for these two questions are incompatible, then they must be applied sequentially. Gist is processed more quickly (Brainerd et al., 1999), and so we assume the person first queries the gist and if this fails to produce acceptance, then the person queries the verbatim property. Using Equation 4, we obtain the following prediction

$$EOD(x) = \langle\psi|M_{xG}\cdot M_{xV}|\psi\rangle + \langle\psi|M_{xV}\cdot M_{xG}|\psi\rangle - |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2.$$

The term $\langle\psi|M_{xV}\cdot M_{xG}|\psi\rangle$ is the inner product between the projection $M_V|\psi\rangle$ on to the verbatim subspace and the projection $M_G|\psi\rangle$ on to the gist subspace. The other term $\langle\psi|M_{xG}\cdot M_{xV}|\psi\rangle$ is the complex conjugate $\langle\psi|M_{xG}\cdot M_{xV}|\psi\rangle = \langle\psi|M_{xV}\cdot M_{xG}|\psi\rangle^*$.

Recall that $|M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2 = \langle\psi|M_{xG}\cdot M_{xV}\cdot M_{xG}|\psi\rangle$ so that

$$\begin{aligned}&\langle\psi|M_{xV}\cdot M_{xG}|\psi\rangle - \langle\psi|M_{xG}\cdot M_{xV}\cdot M_{xG}|\psi\rangle\\ &= \langle\psi|M_{xV}\cdot M_{xG} - M_{xG}\cdot M_{xV}\cdot M_{xG}|\psi\rangle\\ &= \langle\psi|(I-M_{xG})\cdot M_{xV}\cdot M_{xG}|\psi\rangle.\end{aligned}$$

Similarly, we have for the complex conjugate

$$\begin{aligned}&\langle\psi|M_{xG}\cdot M_{xV}|\psi\rangle - \langle\psi|M_{xG}\cdot M_{xV}\cdot M_{xG}|\psi\rangle\\ &= \langle\psi|M_{xG}\cdot M_{xV} - M_{xG}\cdot M_{xV}\cdot M_{xG}|\psi\rangle\\ &= \langle\psi|M_{xG}\cdot M_{xV}\cdot(I-M_{xG})\,|\psi\rangle\\ &= \langle\psi|(I-M_{xG})\cdot M_{xV}\cdot M_{xG}|\psi\rangle^*.\end{aligned}$$

Thus we re-express the quantum prediction as

$$EOD(x) = |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2 + I_{VG}(x),$$

with the interference defined as

$$I_{VG}(x) = \langle\psi|(I-M_{xG})\cdot M_{xV}\cdot M_{xG}|\psi\rangle + \langle\psi|(I-M_{xG})\cdot M_{xV}\cdot M_{xG}|\psi\rangle^*.$$

Recall from analysis that any complex number z can be written in terms of product of its magnitude and phase, $z = |z|\cdot[cos(\phi)+i\cdot sin(\phi)]$, and the conjugate equals $z^* = |z|\cdot[cos(\phi)-i\cdot sin(\phi)]$, so that $z + z^* = |z|\cdot cos(\phi)$. Therefore we can rewrite the interference as follows:

$$I_{VG}(x) = 2\cdot|\langle\psi|(I-M_{xG})\cdot M_{xV}\cdot M_{xG}|\psi\rangle|\cdot cos(\phi).$$

To explain the results, we require the interference to be positive. Intuitively, this means that there must be a positive correlation between the 'gist and then verbal' projection and the 'not gist and then verbal' projection.

The violation of the independence bound shown in Figure 2 requires that

$$I_{VG}(x) > |M_{xV}|\psi\rangle|^2\cdot|M_{xG}|\psi\rangle|^2 - |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2.$$

Consider, for example, the results that produced the largest violation of the independence bound reported by Brainerd et al. (1999). This occurred in Experiment 3 with the related non target probe under the no prime condition reported in Table 2 of Brainerd et al. (1999). The results reported for this condition are as follows: P(RV) = .52, P(RG) = .68, and P(RVG) = .70. For these results we obtain P(RV)·P(RG) = .3536 < EOD(R) = .52 + .68 − .70 .50. The excess over the independence bound equals .50 − .3536 = .1464. To explain these results we need to satisfy the constraint $|M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2 < .3536$ and therefore $I_{VG} > .1464$ so that $|M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2 + I_{VG} = EOD(R) = .50$.

In order to explain those occasions when EOD(x) exceeds P(xVG) as found in Figure 1 of this paper, we must impose a stronger constraint:

$$I_{VG}(x) > (|M_{xV}|\psi\rangle|^2 + |M_{xG}|\psi\rangle|^2)/2 - |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2.$$

This implies that

$$2\cdot(I_{VC}(x) + |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2) > |M_{xV}|\psi\rangle|^2 + |M_{xG}|\psi\rangle|^2$$

which in turn implies

$$I_{VC}(x) + |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2 > |M_{xV}|\psi\rangle|^2 + |M_{xG}|\psi\rangle|^2 - (I_{VC}(x) + |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2)$$

which finally implies

$$EOD(x) > P(x\mathrm{VG}).$$

For example, if $|M_{xV}|\psi\rangle|^2 + |M_{xG}|\psi\rangle|^2 = 1.0$ then $I_{VG} > .50 - |M_{xV}\cdot M_{xG}\cdot|\psi\rangle|^2$ will produce such an effect.

Since the above calculations make no assumptions about the probe, x, the quantum model can explain the EOD effect for test probes taken from all three target sets. Specifically, the quantum model can account for the EOD effect in the case of the unrelated non target set U. As discussed above, this effect cannot be explained by the dual process model. Thus, the quantum model provides a more complete description of the findings from the conjoint memory recognition paradigm.

Conclusions

In the conjoint memory recognition task, the participant is informed that memory probes may come from either a set of previously presented targets or a set of non targets that are meaningfully related to one of targets, or from a set of unrelated non targets. Under verbatim instructions they are asked to accept only targets, under gist instructions they are asked to accept only related non targets, and under disjunctive instructions they are asked to verbatim or gist items. The important fact is that the person knows that the probe cannot be both in the verbatim category and the gist category. The episodic overestimate effect refers to the fact that the probability of accepting a probe when asked if it is a target plus the probability of accepting a probe when asked if it is a related non target is greater than the probability of accepting a probe when asked if it is either a target *or* a non related target. Logically these two probabilities should be identical because verbatim and gist categories are mutually exclusive, and therefore the two individual probabilities should sum to equal the probability of the union. Furthermore, this finding cannot be explained by a recognition memory model based on categorizing probes according to familiarity. Thus these results pose an interesting paradox for memory researchers.

One possible explanation is that participants do not follow instructions and allow implicit categorizations of probes as both gist and verbatim. Then the episodic over distribution effect equals the joint probability. But given the instructions (which state that this joint probability should be zero), this joint probability should be bounded below the product implied by statistical independence. However, strong violations of this statistical independence bound are also found.

In the past, the only satisfactory model for these findings was the dual process model. This model assumes that individuals categorize the probe into one of three states – a correct recollection state, a non recollection but familiar state, and a non recollection and non familiar state. The probe is accepted in either of the first two states. This model predicts that, after adjusting for response biases, the probability of accepting targets under the verbatim instruction equals the probability of accepting targets under the verbatim or gist instructions. This model violates the classic probability OR rule, but it provides a good fit to most of the data. The problem is that this model does not provide any explanation for the episodic over distribution effects that also occur for non related targets. Also this model cannot explain some unusual conditions under which the episodic over distribution effect exceeds the probability of acceptance under the gist or verbatim instructions.

This paper presented a quantum memory recognition model to explain the episodic over distribution effect. The main contribution of the model is to provide a new way to understand the effect of the three different types of retrieval instructions: verbatim, gist, verbatim or gist. The model assumes that memory is represented by a vector in a high dimensional space. The verbatim instruction is represented as one subspace within this space, and the probability of accepting a probe as verbatim is determined by the squared length of the projection of the verbatim subspace. The gist instruction corresponds to a different subspace, and the probability of accepting a probe as gist equals the squared length of the state on the gist subspace. But these two subspaces are incompatible. They do not share the same basis vectors yet they are not orthogonal either. Given that the subspaces are incompatible, the disjunction question must be answered by using a sequence of two projections. To decide whether to accept a probe under the gist or verbatim question, first the person checks the gist, and if this is not accepted then the person checks the verbatim. Then the probability of accepting the probe equals the probability that the person accepts the gist plus the probability that the gist is not accepted but the person then accepts it as a verbatim. This model is capable of explaining (a) the episodic over distribution effect (including the results for the unrelated non targets), (b) the fact that the effect exceeds the statistical independence bound, and (c) the fact that the effect can exceed the probability of the disjunction.

The quantum model presented here to explain memory recognition is formally the same as one previously developed to explain human probability judgment errors (Busemeyer, Pothos, Riccardo, 2009; Trueblood and Busemeyer, 2010). Thus the same model provides an integration of two quite different phenomena from two unrelated cognitive paradigms. This demonstrates the power of a quantum approach to integrate a broad range of findings using a common set of principles.

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